Chapter 1

Epitome Joannis de Monteregio in Almagestum Ptolemaei

The old 1598 inventory from Varmia mentions one book,[i] "Epithome Joan. De Regiomonte" among other works[ii] on mathematics.[1] We know that aside from the autograph there were no copies of the treatise until 1494.[iii] We also know that copies after 1494 were very rare.[iv] Two years later copies were not needed, because the first edition was published in Venice in 1496.[2]

The first edition is now very rare. I have seen only six copies, three of them at the Jagiellonian Library,[v] the fourth in a private Polish library,[vi] the fifth in the chapter library at Strengnäs (Sweden, Södermanland County),[vii] and the last is at the University Library in Uppsala.[viii]

The Uppsala copy came from the Jesuit college library of Olomouc, as a note on the title page shows.[ix] The copy is a folio volume without a title.[x] Folio 1 (fol. a1) has a title

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[1] The book appears in the section entitled *In sexto ordine sunt Astronomi et Geometriae*. The entry lacks the note "manuscr." always added when the mentioned work was a manuscript, a fact that indicates that the entry referred to a printed book and not a manuscript.

[2]Details about the publication will be given later. Regiomontanus had donated the autograph to Cardinal Bessarion. After the Cardinal's death (Ravenna, 1472), it went together with his other books to St. Marks' Library in Venice, also a well-known fact. The above-mentioned circumstances, the preface, and epigram provide the explanation why the first edition appeared in Venice.

written in large Gothic script: "Epytoma Johannis de monte regio In almagestum ptolomei." The verso is blank.[3] Regiomontanus's treatise begins on folio a2 with a title in abbreviations,[4] followed by a dedication.[5] The dedication continues on fol. a3 [p.60]

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[3] The exemplar from Strengnäs has two more pages following the title page, all part of one quire. As mentioned, there are no such pages in the other copies that I have seen. These pages are unnumbered and have no catchwords (I call them "A" and "B"). Page A directly after the title page is blank. The three remaining folio pages (Av, B, and Bv) contain a dedicatory letter by the publisher: "Johannes Baptista Abiosus Neapolis regni ex Balneolo Mathematicorum professor: Artium et medicine doctor : Uerarum scientiarum Speculatoribus Plurimam Salutem Dicit," with a date (fol. Bv) "**(** Finis Nostre Correptionis Anno mundi .6253. Christi uero .1496. die .15. Augusti In vrbe Ueneta." This interesting preface is even more rare than the book itself. Because the publisher J. B. Abiosus was connected with Dominicus Maria Novara, and indirectly with Copernicus, it is worth getting acquainted with its contents. To avoid interrupting the argument, however, I will wait until later to cite a few excerpts from it.

[4] It reads [p. 59]: "Cl. Ptolomei alexandrini Astronomorum principia *is megalin syntaxin* id est in Magnam Constructionem : Georgij purbachij : eiusque discipuli Johannis de Regio monte Astronomicon Epitoma." [All citations are corrected according to the facsimile edition of the *Epitome* in Joannes Regiomontanus, *Opera collectanea*, ed., Felix Schmeidler, *Milliaria* X, 2 (Osnabrück: Otto Zeller Verlag, 1972), pp. 55-274. Note that the facsimile does not contain the dedicatory letter by Abiosus—*AG*].

[5] The dedication reads [p. 59]: "Reuerendissimo in christo patri ac domino domino Bessarioni : episcopo Tusculano : sancta Romane ecclesie Cardinali : patriarche

concluding with a ten-line epigram: “Io. Lu. Ad Lectorem,” which obviously does not come from Regiomontanus. I surmise that the abbreviated name of the epigram’s author refers to Joannes Lucilius Santritter, who occupies an important place in the history of printing, [xi] especially in Venice.[6]

Constantinopolitano Johannes germanus de Regio monte se offert deuotissimum. Admiranti mihi sepenumero….”

[6] On the back of the page [p. 62] there is a large wood engraving depicting the sky with stars, the sun and the moon, over a huge armillary sphere and underneath it two men and the following words: “PTOLEMEUS” and “JOHANNES DE MONTE. R.,” all in an ornamental frame. The text proper starts on fol. a4 recto [p. 63] with: “Liber Primus Uniuersalis ambitus totius Terre ad totum Celum considerationes que necessario presupponende erant premittit. Theoremata quoque que ad sphericas demonstrationes premittuntur enarrat. Chordarum atque et arcuum tradit doctrinam. Ascensiones demum recte sphere inuestigat.”

At the end of the volume (fol. p7 verso [p. 274]) are the following words: “Explicit Magne Compositionis Astronomicon Epitoma Johannis de Regio monte. Impensis non minimis : curaque et emendatione non mediocri virorum prestantium Casparis Grossch : et Stephani Roemer. Opera quoque et arte impressionis mirifica viri solertis Johannis hamman de Landoia : dictus hertzog : felicibus astris expletum. Anno a prima rerum etherearum circuitione .8480. Sole in parte sextadecima virginis gradiente. In hemispherio Ueneto. Anno salutis .1496. currente Pridie Calendae Septembris Uenetijs : Maximiliano Romanorum rege primo faustissime imperante,” followed by the printer’s trade mark. The volume has no pagination; there are the usual catchwords at the bottom

For the sake of brevity, I shall use the abbreviation "Epit." when mentioning this incunabulum. The second and last edition of this treatise is entitled "Joannis de Monte regio et Georgii Purbachii epitome in C. Ptolemaei magnam Compositionem, continens propositiones et annotationes quibus totum Almagestum . . . dilucidâ et brevi doctrinâ, ita declaratur et exponitur. . ." Basileae, apud Henricum Petrum, 1543. Because it was published in the year that Copernicus died, a more detailed description here would not be useful.[xii] I shall prove with great accuracy, however, that our astronomer took a lot of information from the *Epitome*, sometimes even borrowing whole phrases. He must have had a copy of the first edition but none of the six copies that I mentioned above.

A comparison of the two texts will show the close dependence of *De revolutionibus* on the *Epitome*. I shall then show that Copernicus possessed a copy of the *Epitome* (1496) before 1508, or even before 1504. It is likelier that he already acquired it during his first journey to Italy (1496-1500) but certainly no later than his second journey. Until his own copy is discovered, we will not be able to say whether he acquired it in Padua or nearby Venice where it was printed, or already in Bologna between the fall of 1496 and the spring of 1500. There is, however, significant circumstantial evidence that Copernicus became acquainted with the *Epitome* already during his stay in Bologna. I shall return to this conclusion later in the chapter after proving my assertions about his acquaintance with the work.

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of the recto of each page, in well-defined Gothic print in one column, with numerous wood engravings depicting geometrical figures in the text. Because of its rarity and, as we shall see, great importance in Copernican studies, I felt obliged to provide a detailed bibliographical description of the printing.

Ptolemy's *Almagest* was Copernicus's major source, quoted ninety-nine times in *De revolutionibus*. Although the *Epitome* is an abstract of the *Almagest*, the *Epitome* contains many details and observations not contained in the *Almagest*. For these reasons it is necessary to compare all three texts.

There were two Latin translations of the *Almagest* before the middle of the sixteenth century: a) A translation from the Arabic in the second half of the twelfth century by Gerard of Cremona,[xiii] first published by Peter Liechstenstein in Venice in 1515 fol. b) A later translation directly from the Greek by George of Trapezunt in the mid-fifteenth century (first published in Basel 1541 fol.).

The existence of two Latin translations could cause confusion except for the fortunate circumstance that Copernicus's "working" copy of the *Almagest* has survived at Uppsala Library.[xiv] Because Copernicus's annotations and calculations, not yet published, cover the margins and inside covers, I call it his "working" copy. The copy, rather soiled in some places, was evidently used often. It was issued in Venice in 1515, and thus it was based on Gerard's translation. In his copy Copernicus made corrections of the translator's barbarian Latin and of typographical errors. I shall henceforth refer to this

copy, also a bibliographical rarity these days, as **AV**.[7]

[7] I quote the text according to the copy in the Jagiellonian Library in Cracow, shelf no. Mathesis No. 1530 fol. At the outset I would like to make it clear that it is impossible to quote all of the phrases, remarks, or observations in *De revolutionibus* that show that they originate from the *Epitome*. This would have required a fourfold increase in the length of the columns. To prove the dependence of two texts it is not necessary to provide such exhaustive comparisons. Even the most detailed and accurate textual criticism proves dependence of two texts by showing the dependence of only a few cases. This may sometimes be devious in literary or historical works, but it is not the case with mathematical and astronomical treatises where the scientific meaning is of primary importance, not stereotypical phrases and expressions that all authors can use equally.

AV	Epit.	Revol.
I. 3. Quo scitur quod celum sit sphericum: et motus eius circularis. ….figurarum velocioris motus ex superficialibus est circulus : et ex corporeis est sphera. Et quia figure plurium laterum que sunt in circulis equalibus : plures habentes angulos sunt eis maiores : est circulus maior figuris superficialibus : et est sphera maior figuris corporeis. Celum igitur est maius. . . . (fol. 3, lin. 15-18)	I. Conclus. 1. Celi figuram esse sphericam: et motum eius circularem…. Nature enim peccatum fugienti vniuersis in rebus commoditas placet quam maxima. Celo igitur cuncta reliqua comprehensuro figuram impressit sphericam omnium capacissimam. (fol. a5, lin. 44-46), [p. 65].	I. 1. QVOD MVNDVS SIT SPHAERICUS …. globosum esse mundum, siue quod ipsa forma perfectissima sit omnium…. siue quod ipsa capacissima sit figurarum, quae comprehensurum omnia et conseruaturum maxime decet. (ed. Thor., 1854, p. 11, lin. 9-11), [Warsaw ed., 1975, p. 8, lin. 18-21].
IX. cap. 1. De ordinibus spherarum solis et lune et quinque stellarum retrogradarum. (These details cannot be found here or in the entire *Almagest*, because both Alpetragius and al-Battani were later than Ptolemy by several centuries).	IX. propos. 1. Sphere celestes quo ordine habende sint ostendere…. Alpetragius autem.., sub Marte Venerem : sub qua Solem : deinde Mercurium statuebat…. Praeterea cum eorum corpora Solis comparatione admodum parua videantur : ita quod antiqui Veneris diametrum visualem referente Albategni Solis sub decuplam ponebant. (fol. kv, lin. 23-55), [p. 192].	I. cap. 10. DE ORDINE ORBIUM CAELESTIVM Alpetragius superiorem Sole Venerem facit et inferiorem Mercurium … Praeterea quod parua sint corpora comparatione Solis, cum Venus etiam Mercurio maior existens vix centesimam Solis partem obtegere potest, vt vult Machometus Aratensis qui decuplo maiorem existimat Solis dimetientem,… (p. 25, lin. 24-25; p. 26, lin. 19-22), [p. 18, lin. 18; pp.18, lin. 40-p. 19, lin. 2].

AV.

I. cap. 9. De scientia quantitatis chordarum partium circuli.

(Paragraphs have no titles corresponding to "Propositio" (*Epit*.) or "Theoremata" (*Revol*.); without any introduction to its topic, the text begins directly with geometrical deductions). (fol.5, lin. 41ff.).

I. cap. 9

(Complete lack of title, as above).

Epit.

I. (Concl. 6). Nunc ad scientiam chordarum feliciter descendamus.

Propositio prima.

Data circuli diametro : latera decagoni : hexagoni : pentagoni : tetragoni : atque trianguli isopleurorum eidem circulo inscriptorum reperire.

Sit semicirculus *abg* supra diametrum… (fol. a7, lin. 4-9), [p. 69].

I. (Concl. 6).

Propositio II.

Data alicuius arcus chorda : nota fiet chorda arcus residui de semicirculo.

Patet ex…. (fol. a7, lin. 41-44), [p. 69].

Revol.

I. cap. 12. DE RECTIS LINEIS QVAE IN CIRCVLO SVBTENDVNTVR

Theorema primum.

Data circuli diametro latera quoque trigoni, tetragoni, hexagoni, pentagoni et decagoni dari, quae idem circulus circumscribit.

Quoniam, quae ex centro dimidia diametri,… (p. 37, lin. 6-9), [p. 26, lin. 13-15].

I. cap. 12.

PORISMA

Proinde manifestum est, [quod] cum alicuius circumferentiae subtensa fuerit data, illam quoque dari quae reliquam de semicirculo subtendit.[xv]

Quoniam….

[p. 26, lin. 13-15].

AV.	**Epit.**	**Revol.**
I. cap. 9. (Complete lack of title, as above). (fol. 5v, lin. 58ff.).	I. (Concl. 6). *Propositio III.* Si quadrilaterum inscriptum circulo fuerit, rectangulum quod sub duabus eius rectangulis diametris continetur : est equale duobus que sub lateribus eius oppositis continentur rectangulis pariter acceptis. Sit circulo abgd inscriptum quadrilaterum…. (fol. a7v, lin. 2-7), [p. 70].	I. cap. 12. THEOREMA SECVNDUM EISAGOGON Si quadrilaterum circulo inscriptum fuerit, rectangulum sub diagonijs comprehensum aequale est eis quae sub lateribus oppositis continentur. Esto enim quadrilaterum…. (p. 38, lin. 14-17), [p. 26, lin. 24-27].
I. cap. 9. (A complete lack of title, as above). (fol. 6, lin. 13ff.).	I. (Concl. 6). *Propositio V.* Cuiuscunque arcus in semicirculo chorda data fuerit : chordam medietatis talis arcus notam fieri. Sit in semicirculo…. (fol. a7v, lin. 35-38), [p. 70].	I. cap. 12. THEOREMA QVARTVM Data subtendente quamlibet circumferentiam, datur etiam subtendens dimidiam. Describamus circulum…. (p. 39, lin. 23-26). [p. 27, lin. 19-22].

AV.	**Epit.**	**Revol.**
I. cap. 9. (Complete lack of title as above.)	I. (Concl. 6). *Propositio VI.* Datis chordis duorum arcuum in semicirculo : cognoscetur et chorda arcus ex his compositi. Sint in circulo…. (fol. a8, lin. 16-19), [p. 71].	I. cap. 12. THEOREMA QVINTVM Rursus cum datae fuerint duarum circumferentiarum subtensae, datur etiam quae totam ex ijs compositam circumferentiam subtendit. Sint in circulo…. (p. 40, lin. 12-15). [p. 28, lin. 1-4].
I. cap. 9. (A complete lack of title; the text in this paragraph begins with the repetition of an already quoted comment (fol. 6, lines 13ff.), which does not appear either in the *Epit*. or *De rev*. The proper considerations begin in **AV**, fol. 6, lines 5ff. (“et dicam si descripte sint in circulo due chorde diuerse : erit proportio chorde longioris ad chordam breuiorem”). The new paragraph does not start with an initial as the previous paragraph did).	I. (Concl. 6). *Propositio VII*. Arcuum inequalium in semicirculo : maioris ad minorem est proportio maior quam chorde maioris ad chordam minoris. Sit in semicirculo arcus .bg. maior arcu .ab. chorda maioris sit .bg. minoris sit .ab. Dico proportionem arcus .bg. ad arcum .ab. esse maiorem proportione chorde…. (fol. a8, lin. 29-35), [p. 71].	I. cap. 12. THEOREMA SEXTVM Maiorem esse rationem circumferentiarum quam rectarum subtensarum, maioris ad minorem. Sint in circulo binae circumferentiae inaequales coniunctae *AB* et *BC*, maior autem *BC*. Aio maiorem esse rationem *BC* ad *AB* quam subtensarum *BC* ad *AB*…. (p. 41, lin. 6-10), [p. 28, lin. 19-23].

AV.	**Epit.**	**Revol.**
VI. cap. 7.	VI. propos. XXVII.	IV. cap. 32.
Est autem proportio circulorum ad diametros sicut proportio trium partium et .8. minutorum et .30. secundorum ad partem vnam. Hec enim proportio est: que est inter triplum et septimam eius : et inter triplum et decem septuagesimas primas partium ipsius fere. et illud est opus Arsamidis secundum opus absolutum. (fol. 68, lin. 10-13).	Proportio denique circumferentie circuli ad diametrum : vt ostendit Archimedes : est minor quam tripla sexquiseptima : et maior quam tripla superpartiens .10. septuagesimas primas. Inter has autem media proportio est trium partium .8. minut. 30. secund. ad vnam partem. (fol. h3v, lin 20-23), [p. 168].	Porro Archimedes Syracusanus in Dimensionibus Circuli prodidit circumcurrentem ad diametrum minorem admittere rationem quam triplam sesquiseptimam, maiorem vero quam triplam superpartientem septuagesimas primas decem. Inter has mediam assumit Ptolemaeus ut trium, scrupula prima octo, secunda XXX ad unum. (p. 305, lin. 12-17), [p. 231, lin. 24-p.232, lin. 4].
(There are, of course, no Arabian observations cited here, for they were made at least seven centuries after Ptolemy).	(All of the Arabic astronomical observations made by al-Battani, Arzachel, and Thebit mentioned in *De revolutionibus* are also in Regiomontanus's *Epitome*. There are, however, a few Arabian observations in the *Epitome* that are not mentioned in *De revolutionibus*).	Numerous astronomical observations of the Arabs: al-Battani, Arzachel, Thebit.

Finally, I will quote the same excerpt in all three texts that proves my statement about mathematical precision. It concerns figures constituting a part of an ancient astronomical observation, which, because of an error once made, entered the autograph of *De revolutionibus*, and appeared in all editions to date.[xvi]

AV.

Et similiter etiam inuenimus in considerationibus Taionis considerationem que fuit in anno secundo annorum Adriani in mense apud egyptios nominato anun in nocte quam sequitur dies vigesimus secundus eius : et fuit stella veneris in mane super plurimum quod est longitudinis eius a sole inuenta posterior stella que est in extremitate ale meridionalis virginis secundum quantitatem longitudinis pleiadum : aut minus illa parum : secundum quantitatem stelle ipsius et fuit visus cursus eius ac si esset ipse declinatus in meridie secundum quantitatem diametri lunaris vnius. Et quod huius stelle fixe apud nos fuit locus in illa hora super .28. partes et medietatem et tertiam partis duodecime leonis : ita quod fuit stelle veneris locus solis per cursum suum medium super .17. partes et medietatem et tertiam partis et tricesimam partis vnius libre. ergo fuit plurimum quod est longitudinis eius matutinalis a cursu medio .47. partes et medietas et pars tricesima partis. (fol. 109, lin 55 – fol. 109v, lin 10).

Epit.
[X, Prop. 1]

… observationes. Taion ille in anno quarto Adriani .19. diebus mensis Atus tertii transactis : in mane diei vicesimi : considerauit Venerem distantem a stella fixa que est in extremitate ale meridiane virginis : secundum quantitatem longitudinis pleiadum : dempto fortasse arcu : cui ipsamet stella Veneris subtenditur. Videbatur enim Venus versus meridiem distare a dicta stella secundum quantitatem diametri lunaris. Et quia secundum numerationem Ptolemei hec stella in quarto anno Adriani fuit in .28. gra. .5. mi. leonis : si addiderimus quantitatem longitudinis pleiadum : scil. .1. gra. .30. mi. veniet locus Veneris ad .20. mi. primi gradus virginis. Sol autem medio cursu suo erat in .17. gra. et .52. mi. libre quare longitudo maior matutina fuit .47. gra. .32. mi. (fol. l_{2v}, lin. 32-42), [p. 210].

Revol.
[V, 20]

Rursus ad maiorem huius rei affirmationem assumit (Ptolemaeus) aliud a Theone obseruatum anno quarto Adriani, diliculo diei XX mensis Athyr, qui erat a natiuitate Christi annus CXIX, quarto Idus Octobris mane,vbi reperta est denuo Venus in maxima distantia partium XLVII scrupulorum XXXII a loco Solis medio existente in partibus CXCI scrupulis XIII.

(p. 365, lines 25-30), [p. 280, lin.27-32; Autograph V, 19, fol. 170r, lin. 16-22].

Comparison of the texts shows, first of all, that they deal with the same ancient observations. They are set forth extensively in **AV** and very concisely in *De revolutionibus*.[xvii] Only the year in which the observation was made is important here. If we compare these versions with the Greek text of the *Almagest* (the Basel edition of 1538), or even better with the last Paris edition,[8] we will see at once that the year in all three versions (**AV**, *Epitome*, and *De revolutionibus*) is erroneous. The same passage is preserved in all the Greek codices, including the Basel edition, which we may regard as a manuscript:[xviii]

> Similarly, in the [observations we got] from Theon, we found that in the twelfth year of Hadrian, Athyr [III] 21/22 in the Egyptian Calendar [127 Oct. 11/12], Venus as morning-star had its greatest elongation from the sun …

There is no version that has "in anno secundo annorum Adriani," as in **AV**, or "in anno quarto Adriani," as in the *Epitome* and as repeated by Copernicus, instead of "in the [twelfth] year of Hadrian."

We find connections of phrases as in the comparisons between the *Epitome* and *De revolutionibus* along with the information and observations, including among others the Arabian observations, in all of the books of Copernicus's *De revolutionibus*.[xix] All of this proves clearly that our astronomer was well acquainted with Regiomontanus's treatise before he began studies and calculations that had to be based on Arabian observations. The entire letter to Wapowski and especially its last paragraph[xx] prove that the prodigious

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[8] The Paris edition considers the variants of five different manuscripts as well as the variants in the 1538 edition.

geometrical constructions of Book III based to a large extent on Arabian observations (al-Battani, Arzahel, Thebit) were ready in the first half of 1524.

As for the dating of his acquaintance with the *Epitome*, however, we have to go back much further. Copernicus's *Commentariolus* already contains astronomical facts and phrases that could come only from the *Epitome*. I will prove later[xxi] that, in despite of the generally accepted opinion, Copernicus wrote the *Commentariolus* before 1515 and, most probably, even before 1512.[xxii]

Below are examples of passages from the *Commentariolus* with parallel passages from the *Epitome*:

1. Nunc longo temporis tractu deprehensum est talem telluris positionem ad faciem firmamenti mutari, propter quod ipsum firmamentum aliquibus motibus ferri plerisque visum est, lege nondum satis deprehensa.

(*Comm*., p. 9, lines 10-13), [Zekl ed., p. 12].

The passage is clearly a brief summary of the short Propositio VI (not present in the *Almagest*): "De motu stellarum fixarum quid alij senserint explanare" from *Epitome* VII, fol. h8 [p. 177], where Regiomontanus comments on among other things the supposed dual motion of the eighth sphere mentioned by Thebit (*motus trepidationis*) and later in an altered form by Alfonsus. The next example shows the dependence even more clearly:

2. Dum enim ipsi mundi cardines in centenis annis uno gradu mutabantur, quemadmodum Ptolemaei aevo repertum est, erat tunc anni quantitas, quae ab ipso Ptolemaeo tradita est. Quando autem subsequentibus saeculis potiori mutabilitate moverentur motibus inferioribus obviantes,

tanto brevior annus factus est, quanto translatio cadinum esset maior.

(*Comm*., p. 10, lines 10-15), [Zekl ed., p. 14].

The information about the precession exceeding 1^{o} in the centuries following Ptolemy (*in centenis annis uno gradu*) refers to the discovery by al-Battani of the precession of the equinoxes by 1^{o} after already sixty-six years. Copernicus mentions al-Battani by name a few lines earlier in the *Commentariolus* in close connection with the same matter, namely the length of the tropical year. Because I will have to return to this detail in a while, I only for the moment draw attention to the fact that the *Epitome* (Propositio 2, Book III, fol. c3v-c4) discusses the same topic in the same context and especially in Propositio 6, Book VII (cited above): "… diligentissimum philosophie[xxiii] Albategni … stellas fixas considerauit : et loca earum eis quibus in tempore preterito videbantur, conferebat…. Si itaque ex hoc tempore … dederimus portionem : videbitur stella ipsa in .66. annis solaribus fere mota per vnum gradum…." (*Epitome*, fol. h8, lines 3-15), [p. 177]. A few lines later we find the following: "Siue igitur instrumentorum incertitudo hanc varietatem immiserit: siue motum quendam adhuc nobis occultum stellis fixis natura indiderit : difficile admodum est et erit : huius motus qualitatem eniti : propter tarditatem eius…." (ibid., lines 23-26), concerning the insufficiently explained motion of the eighth sphere, which Copernicus expresses briefly: "lege nondum satis deprehensa" (in passage 1 above).

Two other examples prove the influence of the *Epitome* on the *Commentariolus* clearly:

3.

Epitome

Anni quantitatem per obseruationem elicere.

Diuersi diuersas circa anni quantitatem considerationes habuere. Vetustissimi enim egyptiorum annum solarem reditionem solis ad aliquam stellarum fixarum esse dicebant. Inueneruntque id fieri in .365. diebus : quarta diei : et .130. parte diei. Verum hec anni assignatio non conuenit…. (III, prop. 2, fol. c3, lines 38-43), [p. 97].

Commentariolus

Rectius igitur agit, quicumque annuam aequalitatem ad stellas fixas referet. Quemadmodum … fecimus invenimusque annum 365 dierum et sex horarum ex sextantis fere unius horae semper fuisse, qualis etiam in Aegyptiaca antiquitate reperitur….

(p. 10, lines 17-20), [Zekl ed., p. 14].

In the *Epitome* Regiomontanus took the information about the length of the sidereal year from al-Battani, who gave the same number exactly.[xxiv] The Arabian astronomer says clearly that the ancient Egyptians and Babylonians adopted such a length for the sidereal year. Regiomontanus omitted the Babylonians, and mentions only the Egyptians. Notice that Copernicus repeated after him, omitting the Babylonians and mentioning only the Egyptians. This part of the *Commentariolus*, however, has reached us with some defects.

4.

Epitome

Ideo Hyparchus et Ptolemeus dixerunt annum esse reditum solis in aliquod punctum equinoctij aut solsticij…. Verum … vix potest vera anni quantitas inueniri … reperire poterimus. Hinc Hyparchus reperit annum .365. dierum : et quarte vnius. Ptolemeus vero .365. dierum et quarte vnius minus .300. parte diei … Deinde Albategni … reperit … minus .106. parte diei : que est .13. minuta hore : et tres quinte vnius minuti … Posuit igitur annum solarem .365. dies .5. horas .46. minuta : et duas quintas vnius.

(fol. c3, line 45-c3v, line 45), [pp. 97-98].

Commentariolus

… multis experimentis observationum diversa reperta est. Hanc Hypparchus 365 diebus cum quadrante unius diei,[xxv] Albategni vero Chaldaeus reperit talem annum ex 365 diebus, 5 horis, 46 minutis, hoc est 13 minutis et 3 quintis (here some words, probably a whole line,[xxvi] are omitted in the copy—*LAB*) sive triente unius minuti Ptolemaico breviorem. (p. 10, lines 1-5), [Zekl ed., p. 14].

This part of the comparison is conclusive. Here either Regiomontanus or the publisher of the edition, Abiosus, made a mistake, stating that Hipparchus adopted 365¼ days as the length of the tropical year. Every historian of astronomy knows well that one of Hipparchus's main claims to glory was his proof that the length of the tropical year is smaller than 365¼ days by nearly 1/300 of a day. They also know that Ptolemy, wanting to clear up any remaining doubts about the matter, made new observations of the equinoxes three centuries later, and obtained the same number, actually confirming it. This mistake or oversight in the printed text of the *Epitome* is very characteristic, and at

the same time it is important for considerations about the time when the *Commentariolus* was written. In this connection it is necessary to make the matter absolutely clear by quoting Hipparchus's own words to prove that the mistake actually appeared in the *Epitome*. Because Copernicus used only **AV** (reasonably freeing me from quoting the Greek text), I quote the Latin text.[xxvii]

Ptolemy quotes mostly verbatim many fragments of the now lost work of Hipparchus in his long chapter 1 of Book III. The three following excerpts suffice for our purposes:

a) "Ex eis vero que declarauit Abrachis : iam demonstratum est nobis quod quantitas temporis anni : qui incipit a punctis equalitatis et conuersionis donec ad ea redeat : minor est quarta addita supra .365. dies…"[9][xxviii]

b) "Et similiter vidimus Abrachim concessisse hoc. Ipse enim dicit in libro suo de quantitate longitudinis anni : quod comparauit considerationem tropici estiualis : que fuit… vbi dixit quod ostendit quod in .145. annis festinat tropicus ante superfluitatem quarte per medietatem diei et noctis.[xxix] Ipse quoque in libro suo de mensibus et diebus (postquam premisit sermonem) dixit : 'Secundum vero quod dixerunt midan et attamin[xxx] est longitudo temporis anni .365. dies et quarta et vna pars .76. partium et medietas diei vnius, sed secundum quod dixit felis[xxxi] est .365. dies et quarta tantum.' Post hoc quoque dixit : quemadmodum narrabo verbum ex verbo. 'Nos autem iam inuenimus menses integros contineri a .19. annis : quemadmodum inuenerunt illi. Longitudinem vero anni

[9] **AV**, fol. 27v, lines 30-32. These are, however, Ptolemy's words, not those of Hipparchus (*Abrachis* in Arabic). Hipparchus's words quoted as the conclusion by Ptolemy are more important for our purposes.

inuenimus iam minorem quarta, per vnam .300. partium diei vnius et in trecentis annis desunt sermoni midan quinque dies. et sermoni felis deest vnus dies."[10]

As if this were not enough, Ptolemy quotes a few lines later another fragment from a no longer extant last work of Hipparchus, which he precedes by only one sentence of his own:[11]

c) "Ubi autem aggregantur eius (abrachis) sententie in libro suo dixit quemadmodum narrabo : 'Jam scripsi vnum librum de longitudine temporis anni : in quo declaraui : quod annus solis est tempus in quo incipit motus solis a tropico ad tropicum sui similem : aut ab equalitate sui similem quod continet ex diebus .365. dies et minus quarta diei fere per vnam .300. partium diei vnius cum nocte sua. Neque est sicut estimant disciplinales[xxxii] : vt sit augmentum supra .365. dies quarta diei integra.' Estimo autem iam declaratum esse …"[12]

All of the above allows us to draw the following conclusions:

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[10] **AV**, fol. 28, lines 45-56.

[11] **AV**, fol. 28, line 56-fol. 28v, line 1.

[12] There is yet a fifth text similar in both the *Epitome* and the *Commentariolus*. They are so similar that the text in the *Commentariolus* must be a summary of Regiomontanus's own critical observations about Ptolemy's theory of the moon. Because the matter is closely related to another of major importance, I will quote it later to avoid quoting it twice. This evidence not yet produced will provide final confirmation of the correctness of our conclusions.

1. While writing the *Commentariolus*, Copernicus repeated the same mistake that Regiomontanus made in the *Epitome*; thus he must have known the treatise while working on the *Commentariolus*.[13]

2. While reading chapter 1 of Book III of the *Almagest* (not the *Epitome*), only a little attention was needed even in Gerard's barbaric translation (that is, **AV**, from which the quoted excerpts above come) to notice and distinguish Ptolemy's proper comments from quotations of Hipparchus. It is obvious, then, that Copernicus did not yet know the *Almagest* itself while he was writing *Commentariolus*, and took the ancient observations and other information that he needed from the *Epytoma Joannis de Monteregio in Almagestum Ptolomei*.

As it turned out, this was an advantage for the history, verification, and success of Copernicus's great discovery. Although the *Epitome* did not repeat all of the ancient observations provided in the *Almagest*, it included numerous, valuable, medieval

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[13] [As we saw above] this mistake concerns the length of the tropical year, 365 days, 5 hours, 55 minutes, and 12 seconds, discovered by Hipparchus, that the *Epitome* and the *Commentariolus* attributed to Ptolemy. Moreover, the *Epitome* attributed to Hipparchus the length of 365¼ days, which he considered erroneous. This mistake must have been due to a misunderstanding of what Regiomontanus said, for in writing the *Epitome*, he did not always distinguish clearly what Ptolemy himself said in the *Almagest* from what Ptolemy quoted literally from Hipparchus. The *Epitome* is a very literal abstract of the *Almagest*, and quotation marks were not yet used in the fifteenth century. Regiomontanus did not use quotation marks, nor even "Haec ille" always used in old treatises instead of quotation marks.

(Arabian) observations, which the Alexandrian astronomer's *Syntaxis*, older by seven centuries, could not have mentioned. It is thus clear that Copernicus wrote the *Commentariolus* before he obtained **AV**, [that is, his copy **AVU**—*AG*]. The editio princeps is dated 10 January 1515.

The above reasoning leads us inevitably to the conclusion [that Copernicus relied on the *Epitome* for the writing of *Commentariolus* and *De revolutionibus*—*AG*].

[The Dating of the *Commentariolus* and relation to the *Epitome*—*AG*][14]

The *Commentariolus* was not and could not have been a work announcing[xxxiii] the publication of the supposedly finished *De revolutionibus*, as the meritorious researcher of Copernican problems and discoverer of the work stated.[xxxiv] The reason is that the system presented in the *Commentariolus*, though heliocentric, is quite different from the heliocentric system in *De revolutionibus*.[15] The two systems are so kinematically different that I am surprised that no one noticed the differences earlier. It certainly took more than a few months to work out in detail the system presented in the

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[14] This is sufficient for the time being for discussing the *Commentariolus*, time that this interesting treatise fully deserves. In another chapter of the present work, moreover, we will find still other features that prove with mathematical precision that Copernicus completed the work not in 1533 or 1539 but definitely before 1515 and most probably before 1512. I emphasize that what I call "very probable" would normally be considered absolutely certain, but I wish to observe accuracy and caution so that conclusions are presented without exaggeration.

[15] I will prove this assertion later.

Commentariolus, ready before 1512. The work was so advanced that Copernicus was able to compare the calculations with his own observations of Mars in January 1512. We have as yet no evidence giving more detailed information about the time Copernicus started the work.[xxxv] Anyway, surely years and not just months must be considered. Note that Copernicus spent the five years before 1512 at the court of the Bishop of Lidzbark, his uncle. Copernicus supervised his uncle's medical care,[xxxvi] published his translation of the letters of Theophilactus, and accompanied the bishop on his journeys.[16] These numerous distractions, not at all desired by someone seeking private time for himself,[xxxvii] indicate that he needed about two or three years to complete a work so complicated and full of elaborate calculations. This period coincides with Laurentius Corvinus's stay in Varmia and his close contacts with Copernicus. In a poem published with Copernicus's translation of the letter of Theophilactus in 1508, Corvinus refers to the "mira principia" that the young Varmian canon, Copernicus, used to explain to Corvinus the "celerem lune cursum alternosque meatus / Fratris (scil. solis—*LAB*) cum profugis [tractat et astra] globis."[xxxviii] Were not the "mira principia" in the *Commentariolus* recently then completed and based on observations contained in the *Epitome*? These facts make the conjecture[xxxix] so probable that I would not be astonished to find among the documents left by Corvinus, if any survived, an unknown copy of the *Commentariolus*.[xl]

I do not intend, however, to multiply conjectures even if justified. For now it is sufficient to emphasize how improbable it is to suppose that Copernicus could have undertaken work on the *Commentariolus* without acquaintance with the earlier

[16] To Malbork in August 1506, to Cracow in 1507 and 1509, and to Poznań in June 1510.

observations reported in the *Epitome* of the *Almagest*.[17] To avoid several contradictions I must accept the possibility that Copernicus had the *Epitome* at least from 1508, although his occupation, reading, and observation during his stay in Italy[xli] suggest that he obtained the book even earlier, that is, in the last years of the fifteenth century. I happened to find a note from the first years of the sixteenth century that mentions Copernicus's name in connection with a disputed astronomical problem.[18] It suggests that in 1504 Copernicus was already using a research method different from that of other contemporaneous astronomers.[xlii] I will show later in this work the close connections between the elaboration of Copernicus's first heliocentric system and the *Epitome* of Peurbach and Regiomontanus.

There is doubtless proof that the book assisted the Frombork philosopher for many years, even after he had the *Almagest* itself when Regiomontanus's compendium might have seemed useless. He used the book about 1531 while working on an advanced part of *De revolutionibus*, the end of Book V. Errors in the ancient observations of Venus and numerous mistakes in **AV** compelled Copernicus to return to the *Epitome*.[xliii]

It will surprise no one, then, that still later toward the end of his life, Copernicus and his guest Rheticus went through this book among others "in Musaeo nostro Varmiae."

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[17] Not even the most ingenious painter lacking paint or brush, nor sculptor lacking plaster of Paris or a block of stone, would be able to express his ideas.

[18] Although very brief, the note indicates that while in Cracow in May 1504, probably on his way from Italy to Varmia, Copernicus gave a monthly date for an approaching extraordinary astronomical phenomenon different from the date fixed by other contemporary scholars.

The *Narratio prima* written in Varmia in the fall of 1539 confirms it. In it appear fragments such as "Regiomontanus noster li<bro> III propositione XIII Epitomes,"[xliv] and just below "ut propositione XIIII eiusdem tertii Regiomontanus tradit,"[xlv] etc. The clearest proof, however, is another selection of the same work where Rheticus quotes verbatim an entire paragraph from the *Epitome*:[xlvi]

> Regiomontanus noster libro V, propositione XXII Epitomes inquit: Sed mirum est, quod in quadratura Luna in perigio epicycli existente non tanta appareat, cum tamen, si integra luceret, quadruplam oporteret apparere ad magnitudinem, quae apparet in oppositione, cum fuerit in apogio Epicycli. Senserunt et idem….

The location in Regiomontanus's text given by Rheticus is correct, namely, fol. f6, lines 10-15.[19] I do not think that anyone supposes that Rheticus quoted this rather long piece from memory. The conclusion is obvious. While writing chapter 7 of *Narratio prima*, Rheticus used both the autograph of *De revolutionibus* and the *Epitome* opened at fol. 6. From it came the literally quoted paragraph placed among arguments proving the truth of Copernicus's system because that part of the work

[19] The words in Rheticus's text come from another version or edition. [The 1496 ed., fol. 6: "Sed mirum est, quod in quadratura luna in opposito augis epicycli existente non tanta appareat, cum tamen, si integra luceret, quadruplam oporteret apparere ad magnitudinem suam: quae apparet in oppositione, cum fuerit in auge epicycli"—*AG*].

dealt with this topic.[20] The fragment of the *Epitome* that Rheticus thought necessary to quote literally concerned the weakest parts in the theory of the moon put forth by Ptolemy, Alfonsus, Peurbach, etc.[xlvii] This was the detail, a logical contradiction of the [axiom about uniform motion around its proper center—*AG*], as Copernicus himself said,[xlviii] that provoked his doubts about the truth of the geocentric system. The aged scholar and his young disciple must have engaged in interesting talks on the reasons that compelled him to set aside Ptolemy's theory and on the considerations that led him to create something better. I have proved in several chapters of this work that geometrical and generally logical considerations came first, not connected initially with observations, and that they dealt mainly with the theory of the Moon. Observations, beginning with the Moon, followed his already formulated doubts. Its short distance from Earth and hence clearly perceptible motion promised the easiest testing of objections. That was the order in which his prodigious research proceeded. Copernicus's spirit and method do not differ greatly from ways of seeking truth today, but it was very different from those of medieval learning with its rather obtuse approach to the study of nature.

Apart from the evidence in *De revolutionibus* and *Narratio prima*, the much earlier[xlix] *Commentariolus* already provides clear evidence of the sequence in Copernicus's research. Rheticus's quotation of Regiomontanus's words is closely

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[20] This fact, though important, is simply another piece of evidence showing that while he was working on his treatise, Rheticus received not only the autograph but also oral instructions from the astronomer himself about the genesis of his work.

connected with the following text from the *Commentariolus* in the section entitled *De Luna*:[1]

> Qui vero per eccentricum circuli fieri hoc arbitrantur, praeter ineptam in ipso circulo motus inaequalitatem in duos inciderunt manifestos errores. Consequens est enim mathematica ratione, quod Luna in quadraturis, dum infima parte epicycli dependet, quadruplo fere maior appareat (si modo tota luceret), quam nova et plena, nisi augmentum et diminutionem magnitudinis sui corporis ei temerarie asserit. Sic quoque diversitatem aspectus facit propter notabilem terrae magnitudinem ad distantiam eius circa quadraturas plurimum augeri.

The words "Consequens est enim mathematica ratione" connect two sentences, the first of which stresses the logical contradiction between Ptolemy's de facto non-uniform motion of the center of the epicycle on the circumference of the eccentric carrying the deferent and his main principle about the uniformity of all celestial motions. The second sentence ("quod Luna in quadraturis") is nearly a verbatim repetition of Regiomontanus's words.

Further argument should not be necessary, but whoever does not consider this satisfactory could continue reading what Rheticus writes just after this quotation:

> Sed et D<ominum> praeceptorem meum experientia docuit diversitates aspectus, et quantitates corporis Lunae, in omni ipsius a Sole distantia parum vel nihil differre ab iis, quae in coniunctione et oppositione

contingunt, ut manifestum sit Lunae minime talem, ut receptum, eccentricum tribui posse.[21]

Note that both Rheticus and the *Commentariolus* mention the same ridiculous explanation offered by those who "temerarie" supposed that the Moon decreased and increased in its size. In both texts "diversitas aspectus" refers to "parallax."

If we now open *De revolutionibus* to Copernicus's conclusion of his study of the Moon's motion and its parallax (IV, 27)[22], we read the following:

CONFIRMATIO EORVM QVAE LVNAE PARALLAXES SVNT EXPOSITA. Cap. XXVII. Quod igitur parallaxes Lunae sic expositae conformes sint apparentijs, pluribus alijs experimentis possumus adfirmare: quale est hoc quod habuimus Bononiae septimo Idus Martij post occasum Solis anno Christi MIIID. Considerauimus enim quod Luna occultatura esset stellam fulgentem Hyadum, quam Palilicium vocant Romani: quo expectato vidimus stellam applicatam parti corporis Lunaris tenebrosi iamque delitescentem inter cornua Lunae in fine horae quintae noctis,…[23]

If we now consider the fact that on March 9, 1497, the moon was in quadrature (the first quarter), there is no doubt that this observation was made to check the unavoidable, geometrical consequences of Ptolemy's hypothesis about the eccentric in his lunar theory. In other words, the point was to determine whether the value of the

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[21] *Narratio prima*, Thorun ed., 1854, p. 459, lines 26-29; [*Receptio*, p. 17, lines 21-24].

[22] Followed in IV only by practical methods, among others, on the calculation of the time of conjunctions, oppositions, and eclipses.

[23] *De rev.*, Thorun ed., 1854, p. 297, lines 10-18; [Warsaw ed., p. 225, lines 7-14].

lunar parallax in quadrature adopted in the *Almagest* and the *Theoricae novae planetarum* could really be so big (89') that without any change in size the disk of the moon would have to increase four times. The results of the observation made evident the logical contradiction between's Ptolemy's theory and the [axiom about uniformity—*AG*]. Ptolemy, passing over in silence the actual non-uniform motion of the epicycle on the circumference of the eccentric, adopted its uniformity, thus deserting the main principle that he adopted at the beginning of his work. To Regiomontanus, who failed to perceive the source of the error, the consequences seemed merely a "mirum." The discovery of the contradiction in Ptolemy's theory and those of his successors (Alfonsus, Bianchini, Peurbach) and the explanation of its source proved Copernicus's penetrating criticism as a philosopher and his great proficiency in mathematics, especially geometry. This achievement required neither his nor anyone else's astronomical observations.

Copernicus began his study of the *Epitome* with just such an independent evaluation of the logical basis of contemporaneous astronomy. [First, he reflected on] Regiomontanus's somewhat naive astonishment ("mirum") over the fact that the disk of Ptolemy's Moon in quadrature was normal in size even though according to the *Almagest* and the *Theoreicae novae planetarum*, etc., it should be four times larger. Copernicus did not need astronomical observations to prove this wrong. This paragraph from the *Epitome*, then, did not provoke him to examine the consequence of the ancient theory because its illogicality was obvious to anyone who was not already misled by the supposed periodic change in the size of the Moon. [Rather,] it provoked him to examine the basis of Ptolemy's theory, and to solve by means of observations

the problem of the [supposed] existence of the fictitious eccentric, even if someone "temerarie" wanted to believe in the periodic change in the size of the Moon. To measure the lunar parallax (and thus its distance from Earth), it was sufficient to state whether this supposed, very doubtful reduction of distance (nearly by half) in quadrature is true or not. The reply is in IV, 27 of Copernicus's immortal work. Copernicus conducted the observations of the Moon in Bologna on March 9, 1497, to confirm his reasoning, for he had already concluded by such reasoning that there is a logical contradiction in the ancient lunar theory. After that night the following statement was fully justified:[li] "coelum ipsum veraci testimonio."

All of these circumstances enable us to conclude that Copernicus had the *Epitome* of Peurbach and Regiomontanus no later than the beginning of March 1497. He made the observation in Bologna, where the young canon and astronomer began studying in September 1496. In January of that year, the *Epitome* appeared, just printed in Venice, with both its "mirum"[lii] and numerous Arabian observations never before published, as if in time for Copernicus's arrival in Italy.[24]

Because Copernicus had the *Epitome* very early, its contents must have influenced the formation and development of his astronomical ideas in many ways at a very early stage.

[Excursus on the publisher's dedicatory letter—*AG*]

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[24] I do not intend to try to settle at the present time whether Copernicus discovered the logical contradiction in the ancient theory in Bologna for the first time, or arrived there already with this idea.

The introduction by the publisher J. B. Abiosus, possibly available at the present time in only one copy (Strengnäs), contains several details of interest along with some superstitions. I will quote the most important excerpts.

He begins with a discussion of astrology (where he refers to the arithmetic of Boethius among others) concluding "Sed ad nostrum dialogum de ipsius (sc. Astrologiae) defensione[liii] remittimus. Ipsa namque verissima est scientia…" Then follows a further discussion on the same subject (an apologia for astrology), and below:

> Pythagoras ait : de omni re in vtramque partem disputari posse. Nausiphases ait : ex his que videntur esse : nihil magis esse quam non esse. Parmenides ait : ex his que videntur nihil esse ab vniverso zenocleantes (sic!) omnia negocia de negocio deiecit : ait nihil esse… Democritus animam igneam opinatus est : Diogenes aëream atque Anaximenes…..Thomas aquinas creari a deo animam opinatus est ; ipsamque corporis formam esse. Averrois vero intellectum vnum toti humanae speciei immissum putauit….[liv]

He concludes the polemic with the following words from Averroes's commentary on Aristotle's *Metaphysics*: "Sic etiam motus celestes demonstratione comperti veri non essent," and then he deals with problems more closely connected with the book. On fol. Bv, line 12, the publisher writes:

> Quapropter cum per orbem nostris laboribus librum librorum Astrologie Johannis alemani de monte regio : ac Georgij purbachij sui preceptoris comperierim : ipsumque manibus meis scripserim. Ac etiam vt tantus sapientie thesaurus non latitaret : ipsumque scriptorum incorreptionibus (!) et figurarum transpositionibus : et multarum litterarum erroribus emendaui :

> Imprimendumque largitus fui Gaspari scilicet atque Stephano qui impressorum errores optime correxerunt. Per quem namque librum omnes celorum motus vera demonstratione probantur. Qui namque Epithoma Johannis de monte regio et Georgij purbachij in Almagesto Ptolomei nominatur…

and discusses the superiority of this book over Ptolemy's *Almagest*. Among other things, he emphasizes that in the *Epitome* Ptolemy is quoted in Latin, so that it is not necessary to use the *Almagest* in Greek, nor must one learn Greek to read it.[25]

[Conclusion—*AG*]

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[25] He then writes that he is printing the work because the art of printing has been newly discovered to save many things on earth from destruction, including books in connection with the approaching disaster: "…aetas nostra…librorumque impressionem comperit : quorum multitudo portendere videtur futuram maximam mundi corruptionem. Quare sagax natura tot et tantos libros per mundum disseminare voluit : vt in climatum corruptionibus et scientiarum iactura anno Christi .1503. et .1524. et in alijs futuris proximis coniunctionibus penitus non amittantur. Sic quoque Gaspar vir pius huius almi operas impressor : et impressorum errorum diligens emendator : scientiarum pietate commotus : hoc opus sanctum almumque imprimere voluit. Ego quoque Johannes Baptista Abiosus alia multa dignissima opera Johannis de Regiomonte sapientissimi viri imprimenda largiar : librum scilicet triangulorum : et etiam probleumatum : et instrumenta : vt celestes motus vberrime quilibet intueri : obseruare : intelligereque recte valeat." The promises were not fulfilled, and Regiomontanus's trigonometry appeared in Nuremberg thirty-seven years later.

I take the opportunity to quote a few excerpts from the *Dialogue* of the same publisher, as it is very characteristic for those times of the last vain efforts to save the cause that could no longer be saved. The dialogue is between Ptolemy and a "sophist," who expresses numerous and serious objections against contemporaneous astronomy.

> Ptolemeus…. Et si planetarum motus agitari possunt propter octave spere motum non causatur sensibilis error pauco tempore et eo magis quam Alfonsus maximam planetis veritatem adiunxit, consideravit namque octave spere trepidationem circa Arietis et Libre principia et iam lapsi sunt multi anni quibus et eclipses [sic] luminarium comprehenduntur et reliquorum planetarum motus veri, et si quis error in Martis motu reperitur,[lv] vel per octavam speram, vel per suum incognitum motum regulari possunt…
>
> Sophist. Motus octave spere est ignotus . quidam putaverunt quod super polos mundi, quidam vero polum proprium octave spere Stellam arcticam dixerunt, quo fit planetarum motus falsos esse per accidens iudicemus.
>
> Ptolomeus. Tradidimus namque sapientiam universalem in magna nostra Almagesti compositione, ubi syderum loca innotescunt : Ubi quoque precepimus ut quibusdam annorum interstitiis nova constituatur observatio, ut fragmenta relicta in temporis longo intervallo sensibilem errorem non conferant. Hoc quoque noster successor Joannes de Monte Regio retulit : quod oportet ad fixas stellas sepius habere oculum, ut eius motus errorem nobis non afferat. (fol. b2v)….
>
> Sophista. Quam ergo certitudinem consequi potuistis de motibus celestibus, cum magnus ille philosophus Aristoteles, ac eius Commentator

> Averroes ecentricos et epicyclos negent : ita quod Averroes in desperationem astrologiae sui temporis ruit et cum fuerit senio confectus profitetur numquam potuisse comperire hanc Ptolemei astrologiam esse veram, ut Metaphysice libro patet et in libro de celo et mundo…. (fol. b5v).

Ptolemy's reply as delivered by Abiosus is so incompetent that I will not quote it. The last two folios present the author's 109 "Conclusiones," and prove what an ardent supporter of astrology he was. Two of them are worth our attention.

> Conclusio 18: Uenerem et Mercurium posse Solis portionem obumbrare affirmamus.

The sentence is interesting because it may be connected with an important passage from *De revolutionibus* I, 10 concerning the same problem [a solar eclipse by Venus and Mercury].[26]

> Conclusio 108, penultimate: Perquisitio veritatis an loca astrorum sint vera, non potest per logicam fieri, sed mensuris geumetricis (*sic*) et instrumentis et non per vociferationes sophisticas (fol. e5v).

The passage proves how deeply concealed even from the eyes of its eminent specialists the true, logical cause of the decline of contemporaneous astronomy was. They sought deliverance in auxiliary methods such as instruments, measurements, and observations, and, deceiving themselves and others, hoped that they might rescue it, unaware that nothing could come of it without critical and creative thought.

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[26] Thorun ed., p. 26, lines 15-26. The source of both seems to be Macrobius (*In somnium Scipionis*) or even more likely Martianus Capella (*De nupt. Philol. et Mercurii*).

While these apologias were being written and printed, ideas bearing the coming revolution in the old conception, sanctified only by decrepitude, were developing in the mind of the ingenious twenty-year-old Cracow student. The turning point conditioned Copernicus's creative work, and gradually brought him, freed from contradictions and errors, to his immortal discovery. Through the survival of his many annotations and their proper interpretation, we can now clearly outline the development of Copernicus's thinking. I have devoted this and the next chapters of this work to this topic and to the substantiation of many conclusions, where I discuss the life story of the astronomer from causal and chronological perspectives. In this case I availed myself of the results of my own and other scholars' studies without supplying reference notes.[lvi]

Footnotes

[i] See Franz Hipler, *Analecta Varmiensia* (Braunsberg, 1872), p. 59, line 18-p. 60, line 1.

[ii] Namely among other repositories.

[iii] See further in this chapter, and evidence provided by Santritter quoted in chapter 2 below.

[iv] I know of only two copies: Cod. Ms. 44 in the Imperial Library in Vienna (from the end of the fifteenth century), and Cod. Ms. 595 in the Jagiellonian Library in Cracow from 1496.

[v] The shelf numbers are Mathesis 1320 fol. (bound together after Mathesis 1530 fol.), Inc. 1123 fol., and Inc. 2519 fol. All three are missing the two pages following the title page, containing the preface by the publisher, Joannes Baptista Abiosus, but they are otherwise very well preserved. Nicholas of Wieliczka the elder (d. 1510) once owned the second, as a note on the title page and his other annotations in the text show. The third has a sixteenth-century note: "Collegii maioris."

[vi] Dr. Adam Ostoja Ostaszewski of Wzdów near Rymanów (Galicia) purchased it in Nürnberg around 1890. It has no Varmian shelf number, and contains only the conclusion of the publisher's preface.

[vii] The shelf number is T. 509, fol. This is the only copy that I have seen that contains the publisher's preface, a matter of some interest. The copy has the following note on the title page: "Ex libris olim Hermanni Bulderi mathematici 1612." It could not, therefore, have been the property of the Varmian library, for it was plundered in 1626. I take this opportunity to thank the Episcopal chapter of Strengnäs for sending the book to me in Cracow along with three other old printed books from the same library at the request of

the Cracow Academy of Sciences. I also thank Mr. Henryk Bukowski in Stockholm for his kind intervention in the matter.

[viii] Shelf number 34. V. 33, folio.

[ix] This incunabula edition was for some time at the cathedral library at Linköping, as another note in a different hand shows: "Biblioth. Lincopiensi." The Swedes did not occupy Olomouc until 1642; the Varmian libraries were robbed earlier.

[x] See Hain 13 806.

[xi] See Weidler, *Hist. Astr.*, pp. 287, 295, and especially 328, concerning Santritter as astronomer. He owed his fame mainly to his *Almanach perpetuum* (Venice 1498), which though not truly perpetual demonstrates his extraordinarily speculative mind. At first I connected the *Almanach perpetuum* mentioned in the old inventory of the Varmian Library with Santritter (*Analecta Varmiensia*, p. 60, line 3, without the author's name). The copy now at Uppsala did not come from Varmia, however. Other circumstances (discussed elsewhere) indicate that the Jewish publisher Abraham Zacut v. Zachut, who lived in Salamanca at the end of the fifteenth century, prepared the edition. It had the same title, and was published in Venice in 1472 and 1502 in 4^{o}. See Weidler, pp. 269-270.

[xii] For details, see Delambre, *Histoire de l'astronomie du moyen âge* (Paris, 1819), p. 285 and following.

[xiii] See Baldassare Boncompagni, *Dalla vita e della opere di Gherardo Cremonese traduttore del secolo duodecimo e di Gherardo da Sabbionetta, astronomo del secolo decimoterzo* (Rome, 1851), p. 5, line 20.

[xiv] L. Prowe, who had this copy in his hands, first suggested that it could belong to Copernicus. See Prowe, *Mittheil. Aus Schwed. Archiven und Bibliotheken* (Berlin, 1853), and *Nic. Copp.* II (Berlin, 1883), p. 420, which contains no new details. He merely provided the title of the edition and did not study the annotations in the book. The edition and the notes also escaped the attention of Prof. M. Curtze. For details with completely new information about Copernicus's use of the copy at Uppsala, see chapter 10 of the present work.

[xv] In *De revolutionibus* Copernicus always used *circumferentia* instead of *arcus*, *subtensa* instead of *chorda*. He never used *radius* as the radius of a circle but instead *linea quae ex centro*, sometimes even without *quae*. He always used *ratio* instead of *proportio*. [Almost all of Birkenmajer's quotations of Copernicus and Rheticus cited below were from *Revolutionibus orbium caelestium Libri VI*, ed. Societas Copernicana Thorunensis (Toruń, 1873), usually cited as Thorun ed., but he also referred to the 1854 Warsaw edition, which included editions of *Narratio prima* and other minor works of Copernicus such as the Letter to Wapowski, also referred to as the Letter against Werner. See the Thorun edition, 1873, pp. XV-XVI. I have corrected the quotations from *De rev*., following the modern Warsaw edition, *Opera omnia* II (Warsaw: Polish Scientific Publishers, 1975)—*AG*].

[xvi] Only Nicolaus Mulerius noticed Copernicus's mistake in numerical data concerning this obervation by Theon. Mulierius writes in his comment to the Amsterdam edition of *De revolutionibus, Astronomia instaurata*, 1617, p. 384, lines 10-13: "In secunda observatione Theonis adnotanda, nonnihil dormitasse videtur Copernicus cum eam refert ad annum quartum Adriani, ad annum Christi 119, corrigendi sunt ist numeri. Nam erat

annus Adriani duodeciumus, Christi 127, reliqua consentiunt cum notatione Ptolemaei." Again also on observations, on p. 483, lines 10-12: "Copernicus cap. 20 lib. 5, hanc Theonis observationem refert ad annum Adriani quartum, Athyr 20, Christi119, servato eodem die anni. Sed Ptolemaeus ad annum Adriani duodecimum." He did not, however, guess that what he thought was caused by Copernicus's dozing actually derived from someone else's mistake, namely Regiomontanus's in the *Epitome*. Other details on this topic had to be left to chapter 10 of the present work.

[xvii] The final result given in *De revolutionibus*, 191°13', differs from the value given in the two other texts: 17°52' Librae = 197°52'. I will explain this fact on a different occasion.

[xviii] We are justified in considering the Basel edition of 1538 a manuscript because S. Grynaeus based it on Cardinal Bessarion's manuscript given to Regiomontanus, which was later lost. See the preface of Rev. Halma to the Paris edition. [The passage is from *Composition mathématique de Claude Ptolémée*, ed. M. Halma, X, chapter 1 (Paris, 1813), Volume 2, p. 195, lines 1-15. I quote the English translation by Gerald Toomer, *Ptolemy's Almagest* (New York and Berlin: Springer-Verlag, 1984), p. 470—*AG*].

[xix] I could add another dozen or so passages from the *Epitome* and *De revolutionibus*. Among them is the method of calculating the solar apogee (*De rev*. III, 16). Instead of availing oneself of the solstices, as Ptolemy advises, the *Epitome* (III, prop. 14) uses observations of the Sun in Taurus, Leo, and Scorpio. Copernicus carried out the complicated observation of 1515 according to this method, which enabled him to calculate the apogee and the eccentric deferent at the same time. I did not cite this case as proof, because neither Regiomontanus nor Peurbach discovered this method. It is in

the *Epitome* but not mentioned by Ptolemy. It was widely known and discussed by Geber ibn Afflah (*Astr*. Book III), whose work the authors of the *Epitome* quoted frequently. It could then be said that Copernicus took this information from Geber's work directly and not from the *Epitome*.

There are other similar passages in both works. For example, "sitque maxima distantia (solis) *nd*, vt Albategni ponit 1146 partes…." (*Epit*. fol. f5v, line 9) and "distantia maxime solis . . . secundum Albategni fuit 1146 …" (*Epit*. fol. f6v, line 41) and its equivalent in *De revolutionibus*: "… putant apogaei solis a terra distantiam esse partium 1146 … attribuentes haec Arataeo illi philosopho inventori" (that is, Albategnius Arataeus; *De rev*. [IV, 19], p. 282, line 6; [1975 Warsaw ed., p. 213, lines 25-26]). Again: "Nos autem inuenimus arcum … ergo declinatio solis maxima nostro tempore est. 23. graduum. 28. minutorum" (*Epit*. fol. b, lines 8-11, also mentioned by Peurbach in Book I), and Copernicus: "Georgius purbachius anno Christi MCCCCLX partes ut illi XXIII, scrupula vero XXVIII adnotavit" (*De rev*., Thorun ed., pp. 171-172 in a note that was cancelled in the autograph); [cf. *Opera omnia* I, fol. 79r, lines 26-27; *Opera omnia* II, p. 126, lines 23-24, fn. to line 23—*AG*]. The year 1460 (MCCCCLX) in connection with Peurbach proves only that Copernicus was acquainted with his *Theoricae novae planetarum* where the author wrote down "1460" at the end (Venice ed., 1490, fol. f7, line 18) because the year is not mentioned in the *Epitome*.

[xx] "Quid demum ipse de motu non errantium stellarum sphaerae sentiam? Quoniam alio loco destinata sunt, superfluum putavi et impertinens hic amplius immorari. …" (*Inedita Coppernicana*, p. 33, lines 4ff.) I quote this version because the text of this important letter is more correct than in the Warsaw edition of *De revolutionibus* (1854) where it

was published for the first time. [The Letter to Bernard Wapowski is also referred to as the Letter Against Werner. See "Epistola Coppernici Contra Wernerum," *Das neue Weltbild*, Commentariolus, Brief gegen Werner, and De revolutionibus I, ed. and tr. Hans Zekl, [Latin and German] (Hamburg: Felix Meiner Verlag, 1990), pp. 38-57, at 56—*AG*].

[xxi] In chapter 3 below.

[xxii] Not about 1533, as Prof. Curtze put it (cf. *Inedita Coppernicana*, pp. 3-4, and p. 70, abstract 99_2), or, in contradiction with his own opinion (ibid. p. 9, note), as late as 1539 under the influence of Rheticus, as he believed.

[xxiii] The word *indigator* or its equivalent was omitted in the text of the *Epitome.*

[xxiv] Albategnius astronomus peritiss. *De motu stellarum ex obseruationibus tum proprijs*… (Nuremberg, 1537), cap. XXVII, fol. 26, lines 2-6: "Multiplicem dissonamque sententiam in temporis anni quantitate uetustissimos protulisse compertum est. Aegyptiorum etenim et ex Babylonia uetustissimi quidam eam ex 365 diebus et quarta ultraque parte ex 131 diei partibus constare dicebant. Ptolemaeus autem…" The number "131" seems to be more correct than the number "130" given in the *Epitome.* See L. Am. Sédillot, *Matériaux pour servir*…, Vol. II (Paris, 1849), p. 502, lines 13-16; Delambre, *Histoire de l'astronomie du moyen âge*, p. 34. Al-Battani's treatise also has another name, *De scientia stellarum*. Plato of Tivoli (Plato Tiburtinus) translated it into Latin already in the twelfth century. See H. A. Weidler, p. 210; Delambre, *Histoire*, p. 10, and especially B. Boncompagni, *Delle versioni fatte a Platone Tiburtino traduttore del secolo duodecimo*, in *Atti del' accad. pontiff. dei Nuovi Lincei*, 4 (Rome, 1851). Sédillot read 1/131 (perhaps in the Arabic text?) and not 1/130 as in the *Epitome*. Because the Latin text of al-Battani's treatise was first published in 1537 (Nuremberg), it could not have

been the direct source for Copernicus in the *Commentariolus*. Manuscript copies of the Latin version were always very rare, and even today there are only three or four extant. Although Copernicus availed himself many times of the observations carried out by al-Battani, he never mentions the title of the treatise, nor does he provide reference notes to it, something he does as a rule when he cites Ptolemy's *Almagest* in *De revolutionibus* or even in the letter to Wapowski. These facts indicate clearly that Copernicus had to avail himself of second-hand information. A more detailed comparison between *De revolutionibus* and al-Battani's treatise supplies several facts proving that Copernicus did not read the Arabian astronomer's treatise, that is, Plato of Tivoli's Latin translation, either in manuscript or print.

Dr. Adam Ostoja Ostaszewski lent me a copy of the 1537 edition of al-Battani's treatise. He has this rare edition in his rich private collection at Wzdów near Rymanów, which no public library in Poland possesses.

[xxv] The Vienna copy very probably lost a whole sentence, possibly one line, mentioning the length of the topical year that Ptolemy adopted (see the fragment from the *Epitome*). The word *Hanc* before *Hypparchus* in the *Commentariolus* does not fit well with the previous sentence. The nearest word of feminine gender is *observationum* in the plural, and *annuae revolutionis* is too far removed (p. 9, line 23) for *Hanc* to refer to it. If that word were emended to *Hinc*, thus referring to *multa experimenta observat*, then it would fit perfectly. In that case as well, the *Commentariolus* would correspond even better with the *Epitome*.

[xxvi] This conclusion is evident from the identity of the difference of *13 minut et 3/5 minuti* in both texts, and from the fact that the statement in the *Commentariolus* that the fraction 3/5 is equal to the *triens*, that is to 1/3, is impossible.

[xxvii] See chapter 10 of the present work.

[xxviii] The two most important are *perì heniausìou megéthous* and *perì tes metaptóseos ton tropikon kaì hisemerinon semeíon*. [Birkenmajer provides no reference. See *Composition mathématique de Claude Ptolémée*, Halma ed., Vol. 1, p. 163, referring to Hipparchus's treatise on the length of the year. Cf. Toomer tr., p. 139. The second refers to Hipparchus's treatise on the displacement of the stolstitial and equinoctial points. See Halma ed., I, p. 132, and Toomer tr., p. 132—*AG*].

[xxix] Equals half of the day and night (*nukthémeron*).

[xxx] Meton et Euctemon.

[xxxi] Although it may seem improbable, this is identical with Kalippos. See chapter 10 of the present work.

[xxxii] Equivalent to *Philosophi, sapientes*. See chapter 10 of this work on Copernicus's notes in the Uppsala copy of the *Almagest*, to which I refer as **AVU**, [namely Copernicus's own copy of **AV** now in Uppsala Library—*AG*].

[xxxiii] Prof. M. Curtze first called it a "Selbstanzeige eines Buches," (that is, *De revolutionibus*), and others adopted his opinion.

[xxxiv] As is known, Prof. Curtze discovered the work in the Imperial Library in Vienna in 1877. Prof. Arvid Lindhagen discovered the second, more complete copy, once the property of Hevelius, in the Library of the Astronomical Observatory in Stockholm in 1881.

[xxxv] We will see in chapters 2, 3, 6, 7, 10, and 14 of the present work that comparison between Copernicus's own annotations in incunabula that he owned and the autograph of *De revolutionibus* supplies several pieces of such evidence.

[xxxvi] See "Regesta Copernicana," ed. Franz Hipler, *Spicilegium Copernicanum*, pp. 268-269, No. 13.

[xxxvii] See the letter of Tiedemann Giese to George Donner [8 Dec. 1542], *Spicilegium Copernicanum*, p. 352: "…vita incolumi solitudinem amavit…"

[xxxviii] See "A farewell to Prussia" at the beginning (p. 4) of Haller's edition of Copernicus's translation of the letters of Theophilactus Simokata. See the reprint in *Spicilegium Copernicanum*, p. 75, lines 25-30. [The text cited is from *Die Humanistischen Schriften*, ed. S. Kirchner and A. Kühne, *Nicolaus Copernicus Gesamtausgabe*, Vol. 5 (Berlin: Akademie Verlag, 1999), p. 6, lines 33-34—*AG*].

[xxxix] In chapter 14, I prove, based on Copernicus's annotation (see chapter 2) compared with the autograph of *De revolutionibus*, that the theory of the motions of the Earth and Moon was ready no later than the first half of 1509, and that afterwards he changed only numerical data, and completed finishing details.

[xl] The close friendship between the two going back to their days together in Cracow indicates that Copernicus revealed not only the "mira principia" to him, but most probably did justice to his interests by supplying the highly educated humanist with a copy of the paper. Whoever agrees may hope to find the supposed copy of the *Commentariolus* in Wrocław, where Corvinus died in 1527. [Cf. *Die Humanistischen Schriften, Gesamtausgabe*, Vol. 5, p. 6, line 36: "Causas scit miris querere principijs."—*AG*].

[xli] See chapters 2, 5, 6, and 27 of the present work.

[xlii] See chapter 20 of the present work.

[xliii] This assertion is proved in chapter 10.

[xliv] *Narratio prima*, Thorun ed., p. 454, line 4. [The text cited is from *Receptio Copernicana*, ed. H. Nobis et al., *Gesamtausgabe*, Vol. 8/1 (Berlin: Akademie Verlag, 2002), p. 11, lines 35-36—*AG*].

[xlv] Thorun ed., lines 12-13; *Receptio*, p. 12, lines 6-7.

[xlvi] Passage seven, Thorun ed., p. 459, lines 21ff; *Receptio*, p. 17, lines 17-20.

[xlvii] I am referring to the improbably large difference between the largest and smallest lunar parallax, the inevitable, general result in that case of the motion of the epicycle on the eccentric deferent revolving uniformly, but not around the center of the deferent.

[xlviii] *De revolutionibus* IV, 2-3 and V, 2 at the end. Copernicus's perception of that contradiction became in his hands the principal battering ram for demolishing the old astronomical edifice. See chapters 3, 6, and 7 of the present work. Ptolemy deserted the principle that stated that all motions of celestial bodies are either uniform or composed (by turns) of uniform motions. Copernicus was the first to notice it. The contradiction is most evident in Ptolemy's lunar theory, in which the epicycle moves uniformly on the deferent, but not with respect to the center of the deferent, which means that the motion was de facto non-uniform.

[xlix] See chapter 3, where we show that it was written before 1515 because it contains astronomical data established before 1512.

[l] *Commentariolus*, in the supplement to *Inedita Coppernicana*, in *Mittheil. d. Coppern. Vereins* (Toruń, 1882), Vol. 4, p. 6, lines 29-38. [The modern Warsaw edition and the *Gesamtausgabe* have not yet published their editions of *Commentariolus*. The text cited is from *Das neue Weltbild*, Zekl ed. (Hamburg: Felix Meiner Verlag, 1990), p. 18—*AG*].

[li] *Commentariolus* in *Inedita Coppernicana*, p. 10, lines 22-23; Zekl ed., p. 14.

[lii] The second (chronologically earlier), and as far as I know last, remark by Regiomontanus about the lunar "mirum" is in his interesting letter (Rome, February 1464) to John Bianchini in Ferrara. The letter was first discovered in 1786 (published among materials that the Nuremberg Senate managed to save in part after the death of Bernhard Walter). In discussing the discrepancies between planetary theories and the sky, Regiomontanus writes among other things: "In Luna postremo tanta tamque crebra redundant differentia, ut et populares hoc divinum astrorum studium mordaci dente lacerare incipient… Quod si Luna habeat ecentricum et epicyclum, quemadmodum conclamatum est, oportebit Lunam in certo situ quadruplo fere maiorem apparere, quemadmodum conclamatum est, oportebit Lunam in certo situ quadruplo fere maiorem apparere, quam in alio." (Christ. Theoph. de MVRR, *Memorabilia Bibliothecarum*, publ. Norimb. et Univers. Altdorfinae, Part I, Nuremberg 1786, pp. 151-152). I do not need to prove how improbable it would be to assume that Copernicus learned of the "mirum" from this letter instead of the *Epitome* with the above-mentioned passage, which was already in the hands of all astronomers since January 1496.

[liii] This dialogue, very rare today, was printed in Venice in 1494 under the title: *Dialogus in Astrologie defensionem, cum vaticinio a diluvio usque ad Christi annos 1702. Joannis Abiosi Neapolis Regni ex balneolo mathematicarum professoris Artium et meditine* (sic!)

Doctoris, dedicated to Alfonsus the King of Naples and Sicily (in 4^{o}). Because I have a copy of this interesting paper from the Jagiellonian Library (Mathesis No. 8), I have quoted a few interesting excerpts.

[liv] Most of these opinions are taken from Plutarch, *De placitis philosophorum*.

[lv] The biggest differences between the *Alfonsine Tables* and the sky (three or even more degrees) occurred in the position of Mars. The reasons are the nearness of the planet to Earth and the great eccentricity of Mars.

[lvi] I wish to draw attention to the fact that Regiomontanus did not discover the important critical observation concerning the lunar "mirum," although he mentioned them in the *Epitome* and the letter to Bianchini. A manuscript at the Imperial Library in Vienna (the Palatina), No. 5203 (4^{o}), written by one hand about the mid-fifteenth century, contains mathematical and astronomical treatises by Thabit, Jordanus of Nemore, and especially by Peurbach. On folios 100-107 it also contains an anonymous treatise (written or copied in Paris) entitled "De reprobatione eccentricorum et epicyclorum," of great interest and worth publishing as a whole. It begins: "Cum inferiorum cognicio ad celestium conducat inquisitionem…," The problem of interest to us is contained in the words: "Iam venio ad rationes contra illam approbatam scientiam, que si esset vera secundum approbatum sententiam.…sequeretur lunam in plenilunio non plus in sesquialtero esse remociorem, quam in quadraturis i.e. quum est dimidiata lumine, quod est falsum. Quia sic in quadraturis deberet apparere notabilioris dyametri quam in plenilunio, quia si ho (sic!) in longa distantia apparens arcus parue quantitatis in sesqialtero [sic] propinquior poneretur, notabilioris appareret ad sensum omnibus materialis quantitatis, quam antea. Quod tamen de luna non sentimus, quantumcunque diligenter eam inspeximus" (vol. 108v, lin.

7ff.). It ends (fol. 107) with the words: "Sed sufficit eum nunc in generali tetigisse modum. Aug sunt tractatus de reprobacione eccentricorum et epiciclorum .parisius. deo gratias." Whoever the author was (perhaps Nicole Oresme) lived probably in the second half of the fourteenth century, for he mentions the year 1364 as recently passed (fol. 109v, lines 21-22). Although he reveals no knowledge of either the daily or annual motion of Earth, he supplies several details that must attract attention, including his mistrust of the conventional theory of precession (fols. 109-110), the negation of the traditional sequence of planets (the Moon, Mercury, Venus, the Sun…) in the geocentric system (fol. 111, line 14 ff.), and moreover his bold, sometimes even defiant criticism of Ptolemy's and Alfonsus's ideas about the existence of the huge epicycles for Venus and Mars. In his preface to Peurbach's *Tabulae eclipsium* (Vienna 1514 fol.), George Transtetter mentioned certainly the discussed treatise under the same title, and noted that it was once in the library of Andrea Stiborius Boius, a canon and professor at the University of Vienna, who died at an old age about 1515. Cf. Weidler, *H. A.*, p. 331.